

Lecture series in Mathematics

Feb. 21 – 23, 2018, Northeastern University

Aaron Lauda

University of Southern California



This series is part of Spring 2018
RTG@Northeastern activities,
funded by the National Science
Foundation, grant DMS-1645877.



Organizers: Ivan Losev,
Emanuele Macri, Ivan Martino.

Diagrammatic algebra

Wednesday, February 21, 2018, 4-5:30pm.
Room: BK 010.

A new look at quantum knot invariants

Colloquium joint with Math Club
Thursday, February 22, 2018, 4:30-5:30pm.
Room: LA 509.

Higher categories and two-dimensional topology

Friday, February 23, 2018, 4-5:30pm.
Room BK 010.

Diagrammatic algebra

In this talk we will introduce a calculus of planar diagrams that can be used to represent algebraic structures in a wide variety of contexts. We will start by introducing a diagrammatic framework for studying linear algebra. In this framework, familiar notions such as trace and dimension take on a diagrammatic meaning. We will see how the notion of duality transforms algebraic notions into intuitive manipulations of diagrams. Finally, we will see how this diagrammatic reformulation of linear algebra can be used to study invariants of tangled pieces of string (knot theory).

A new look at quantum knot invariants

The Reshetikhin-Turaev construction associates knot invariants to the data of a simple Lie algebra and a choice of irreducible representation. The Jones polynomial is the most famous example coming from the Lie algebra $\mathfrak{sl}(2)$ and its two-dimensional representation. In this talk we will explain Cautis-Kamnitzer-Morrison's novel new approach to studying RT invariants associated to the Lie algebra $\mathfrak{sl}(n)$. Rather than delving into a morass of representation theory, we will show how two relatively simple Lie theoretic ingredients can be combined with a powerful duality (skew Howe) to give an elementary and diagrammatic construction of these invariants. We will explain how this new framework solved an important open problem in representation theory, proves the existence of an (a,q) -super polynomial conjectured by physicists (joint with Garoufalidis and Le), and leads to a new elementary approach to link homology theories categorifying RT-invariants (joint with Queffelec and Rose).

Higher categories and two-dimensional topology

In the second talk we will introduce the world of higher categories. We define the notion of a 2-category and explore some examples that lurk in the background of several advanced undergraduate courses. Our perspective is that 2-categories are really just a framework for studying an enhanced version of the diagrammatic calculus from the previous lecture. We will see how the notion of duality generalizes to the notion of adjunction. By interpreting the resulting diagrams as actual surfaces, we uncover a deep connection between adjunctions (or adjoints) and surfaces. In the process, we will rediscover that certain 2-dimensional surfaces can be completely described using an algebraic structure called Frobenius algebras. Mathematicians usually call this process of turning topology into algebra a "Topological Quantum Field Theory". Our investigations of adjoints in 2-categories will illuminate a simple example of a 2-dimensional (planar) TQFT.